

Mathematical Modelling, Stability Analysis and Control of Flexible Double Link Robotic Manipulator: A Simulation Approach

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Abstract: In present work the various aspects on mathematical modelling, stability and control strategies of flexible double link manipulator have been investigated. A mathematical model of flexible double link manipulator has been developed using lagrangian method. This mathematical model has been characterized using classical and modern control theories. Their time domain and frequency domain analysis has been carried out and our study show that the mathematical model of flexible manipulator is highly unstable. Different control strategies such as PID, LQR and State feedback controller have been implemented for controlling the tip position of flexible double link manipulator using MATLAB programming. State feedback controller uses pole placement approach, while the linear quadratic regulator is obtained by resolving the Riccati equation. The best control strategy for controlling the tip position of flexible double link manipulator is obtained by implementation of LQR controller. Finally, this study proven that LQR control method is the best method as compare to PID and State feedback controller for controlling the flexible link robotic manipulator.

Key words: PID, LQR, MATLAB, State feedback controller

I. INTRODUCTION

The dynamics of nonlinear system can be linearized using classical and modern control theory because it is able to characterize the nonlinear system. The nonlinear differential equations are used to describe the dynamic characteristics of nonlinear systems [1-4]. Practically, nonlinear systems are solved by numerical and graphical methods [5-6]. Nonlinear control is the area of control engineering specifically involved with systems that are nonlinear, time-variant, or both. The response of nonlinear systems to a particular test signals is no guide to their behaviour to other inputs, since the principal of superposition no longer holds [7]. Robotics is one of the fields of study of nonlinear system. Robotics is the art, knowledge base, and the know-how of designing, applying, and using robots in human endeavours. Robotics systems consist of not just robots, but also other [8] devices and systems that are used together with the robots to perform the necessary tasks. Robotics is an interdisciplinary subject that benefits from mechanical engineering, electrical and electronics engineering, biology, and many other disciplines [9]. Robot and robot-like manipulators are now commonly employed in hostile environment, such as at various places in an atomic plant for handling radioactive materials. Robots are being employed to construct and repair space stations and satellites. One type of robot commonly used in the industry is a robotic manipulator or simply manipulator or a robotic arm [8].

Robotic manipulators are mostly used to help in dangerous, monotonous, and tedious jobs. The growing research interest in flexible link manipulators are due to its several advantages such as low energy consumption, high speed and high pay load to manipulator weight ratio makes it suitable for aerospace industrial applications [9-12]. Most of the robotic manipulators are designed and built in a manner to maximize stiffness in order to minimize the vibration of the end-effectors. To have a high stiffness, additional materials have been used for the arm. Hence, the heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating pay load. Many industrial manipulators face the problem of arm vibrations during high speed motion. In order to improve industrial productivity, it is required to reduce the weight of the arms and to increase their speed of operation [9-12]. For these purposes, it is desirable to build flexible robotic manipulators. Due to the importance and usefulness of these robots, a deep investigation for modelling and control of flexible manipulator is required.

In this paper, a dynamic model of the flexible link which has a rotational and translational motion is derived by using a Lagrangian method. The prime objective of this paper is to develop the mathematical model of flexible link manipulators with help of classical and modern control theories and implementation of important control strategies such as PID, LQR and State feedback controller for controlling the tip position of flexible link manipulators through MATLAB[13-15]. State feedback controller uses pole placement approach, while the linear quadratic regulator is obtained by resolving the Riccati equation.

II. RESEARCH METHODOLOGY: Mathematical Modeling of Flexible double Link Manipulators

The dynamic model of flexible manipulator involves modeling the rotational base and flexible link using Lagrange’s method. In order to determine the Lagrangian of system, we need to calculate Potential & Kinetic Energy. The schematic of double link flexible manipulator is illustrated in figure 1. The expression for potential energy and kinetic energy of double link flexible manipulator is given by equation (1) and (2) respectively:

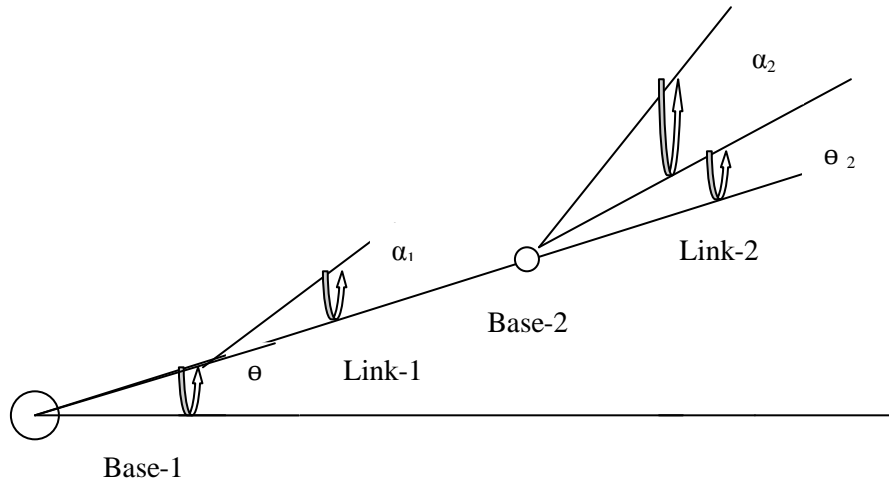


Figure 1: Schematic of double link Flexible manipulator

$$V = P.E. = (1/2) \cdot K_{stiff} \cdot \alpha^2 \tag{1}$$

$$T = K.E. (total) = K.E_{base1} + K.E_{Link 1} + K.E_{base2} + K.E_{Link 2} \tag{2}$$

$$= (1/2) \cdot J_{eq} \cdot \dot{\theta}_1^2 + (1/2) \cdot J_{Link} \cdot (\dot{\theta}_1 + \dot{\alpha}_1)^2 + (1/2) \cdot J_{eq} \cdot \dot{\theta}_2^2 + (1/2) \cdot J_{Link} \cdot (\dot{\theta}_2 + \dot{\alpha}_2)^2$$

Where, $\theta_1, \theta_2 =$ Gear Angles of link 1 and link 2 respectively (radians)

$\alpha_1, \alpha_2 =$ Arm Deflection of link 1 and link 2 respectively (radians)

The assume parameters for double link flexible manipulator are listed in table 1.

Mass of flexible link 1, $m = 0.065$ kg	Length of flexible link 1, $L = 0.3$ m
Mass of flexible link 2, $m = 0.070$ kg	Length of flexible link 2, $L = 0.2$ m
End point arc length deflection, $d = \alpha * L$	Armature Resistance, $R_m = 2.6 \Omega$
Equivalent Moment of Inertia at load, $J_{eq1} = 0.099$ kg.m ²	Links Moment of Inertia, $J_{Arm1} = 0.00195$ kg.m ²
Equivalent Moment of Inertia at load, $J_{eq2} = 0.092$ kg.m ²	Links Moment of Inertia, $J_{Arm2} = 0.000933$ kg.m ²
Equivalent viscous damping coefficient, $B_{eq} = 1.99$	Gear Box Efficiency, $\eta_g = 0.9$
Motor Efficiency, $\eta_m = 0.69$	Motor Torque constant, $K_t = 0.00767$
Back e.m.f. torque constant, $K_m = 0.00767$	Gear Ratio, $K_g = 70$
Link’s natural frequency, $f_c = 3.2$ Hz	Link’s Stiffness, $K_{stiff} = 2\pi f_c * J_{Arm}$
$V_m =$ Armature input voltage	

Table 1: Parameters of double link flexible manipulator

According to the definition of lagrange function, the lagrangian for double link flexible manipulator is given as:

$$L = T - V$$

$$= (1/2) \cdot J_{eq1} \cdot \dot{\theta}_1^2 + (1/2) \cdot J_{Link,1} \cdot (\dot{\theta}_1 + \dot{\alpha}_1)^2 - (1/2) \cdot k_{stiff} \cdot \alpha_1^2 + (1/2) \cdot J_{eq2} \cdot \dot{\theta}_2^2 + (1/2) \cdot J_{Link,2} \cdot (\dot{\theta}_2 + \dot{\alpha}_2)^2 - (1/2) \cdot k_{stiff} \cdot \alpha_2^2 \tag{3}$$

We assume that θ & α are two generalized coordinates.

We therefore have two equations:

$$(\delta / \delta t) \cdot (\delta L / \delta \dot{\theta}) - (\delta L / \delta \theta) = T_{o/p} + B_{eq} \cdot \dot{\theta} \tag{4}$$

$$(\delta / \delta t) \cdot (\delta L / \delta \dot{\alpha}) - (\delta L / \delta \alpha) = 0 \tag{5}$$

Now,

$$\begin{aligned}
 (\delta L / \delta \dot{\theta}_1) &= (1/2) \cdot (2 \cdot J_{eq} \cdot \dot{\theta}_1) + (1/2) \cdot J_{Link1} \cdot 2(\dot{\theta}_1) + (1/2) \cdot J_{Link1} \cdot 2(\dot{\alpha}_1) \\
 &= J_{eq} \cdot \dot{\theta}_1 + J_{Link} \cdot \dot{\alpha}_1 + J_{Link} \cdot \dot{\theta}_1 \\
 (\delta / \delta t) \cdot (\delta L / \delta \dot{\theta}_1) &= J_{eq} \cdot \ddot{\theta}_1 + J_{Link1} \cdot \ddot{\alpha}_1 + J_{Link1} \cdot \ddot{\theta}_1
 \end{aligned} \tag{6}$$

Now,

$$\begin{aligned}
 (\delta L / \delta \dot{\alpha}_1) &= (1/2) \cdot J_{Link1} \cdot 2(\dot{\alpha}_1) + (1/2) \cdot J_{Link1} \cdot 2(\dot{\theta}_1) \\
 &= J_{Link \cdot 1} \cdot \dot{\alpha}_1 + J_{Link1} \cdot \dot{\theta}_1
 \end{aligned} \tag{7}$$

$$(\delta L / \delta \alpha_1) = -K_{stiff} \tag{8}$$

And

$$\begin{aligned}
 (\delta L / \delta \dot{\theta}_2) &= (1/2) \cdot (2 \cdot J_{eq} \cdot \dot{\theta}_2) + (1/2) \cdot J_{Link2} \cdot 2(\dot{\theta}_2) + (1/2) \cdot J_{Link2} \cdot 2(\dot{\alpha}_2) \\
 &= J_{eq} \cdot \dot{\theta}_2 + J_{Link} \cdot \dot{\alpha}_2 + J_{Link} \cdot \dot{\theta}_2 \\
 (\delta / \delta t) \cdot (\delta L / \delta \dot{\theta}_2) &= J_{eq2} \cdot \ddot{\theta}_2 + J_{Link2} \cdot \ddot{\alpha}_2 + J_{Link2} \cdot \ddot{\theta}_2
 \end{aligned} \tag{9}$$

Now,

$$\begin{aligned}
 (\delta L / \delta \dot{\alpha}_2) &= (1/2) \cdot J_{Link2} \cdot 2(\dot{\alpha}_2) + (1/2) \cdot J_{Link2} \cdot 2(\dot{\theta}_2) \\
 &= J_{Link \cdot 2} \cdot \dot{\alpha}_2 + J_{Link2} \cdot \dot{\theta}_2
 \end{aligned} \tag{10}$$

$$(\delta L / \delta \alpha_2) = -K_{stiff} \tag{11}$$

Put equations (15), (16), (17), (18), (19) & (20) in equations (13) & (14), we get;

$$J_{eq \cdot 1} \ddot{\theta}_1 + J_{Arm1} \cdot (\ddot{\theta}_1 + \ddot{\alpha}_1) = T_{o/p} - B_{eq} \cdot \dot{\theta}_1 \tag{12}$$

$$J_{Arm1} (\ddot{\theta}_1 + \ddot{\alpha}_1) + K_{stiff} \cdot \alpha_1 = 0 \tag{13}$$

$$J_{eq \cdot 2} \ddot{\theta}_2 + J_{Arm2} \cdot (\ddot{\theta}_2 + \ddot{\alpha}_2) = T_{o/p} - B_{eq} \cdot \dot{\theta}_2 \tag{14}$$

$$J_{Arm2} (\ddot{\theta}_2 + \ddot{\alpha}_2) + K_{stiff} \cdot \alpha_2 = 0 \tag{15}$$

Where,

$$T_{o/p} = [\eta_m \cdot \eta_g \cdot K_t \cdot K_g (V_m - K_g \cdot K_m \cdot \dot{\theta})] / R_m$$

We will find value of $(\ddot{\theta}_1)$, $(\ddot{\theta}_2)$, $(\ddot{\alpha}_1)$ & $(\ddot{\alpha}_2)$

$$J_{eq1} \cdot \ddot{\theta}_1 - K_{stiff} \cdot \alpha_1 = [\eta_m \cdot \eta_g \cdot K_t \cdot K_g (V_m - K_g \cdot K_m \cdot \dot{\theta}_1)] / R_m$$

$$J_{eq1} \ddot{\theta}_1 = K_{stiff} \cdot \alpha_1 + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g V_m] / R_m - [\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m \cdot \dot{\theta}_1] / R_m - B_{eq} \cdot \dot{\theta}_1$$

or,

$$\ddot{\theta}_1 = [K_{stiff} \cdot \alpha_1] / J_{eq1} + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g V_m] / R_m \cdot J_{eq1} - [(\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_1] / R_m \cdot J_{eq1} \tag{16}$$

Put value of $\ddot{\theta}_1$ in equation (13);

$$J_{Arm1} \cdot \ddot{\theta}_1 + J_{Arm1} \cdot \ddot{\alpha}_1 + K_{stiff} \cdot \alpha_1 = 0$$

$$J_{Arm1} \cdot \ddot{\theta}_1 + J_{Arm1} \cdot \ddot{\alpha}_1 = -K_{stiff} \cdot \alpha_1$$

$$J_{Arm1} \cdot [K_{stiff} \cdot \alpha_1] / J_{eq1} + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g V_m] / R_m \cdot J_{eq1} - [(\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_1] / R_m \cdot J_{eq1} + J_{Arm1} \cdot \ddot{\alpha}_1 = -K_{stiff} \cdot \alpha_1$$

or,

$$J_{Arm} \cdot \ddot{\alpha}_1 = -K_{stiff} \cdot \alpha_1 [1 + (J_{Arm1} / J_{eq1})] - J_{Arm1} [\eta_m \cdot \eta_g \cdot K_t \cdot K_g V_m] / R_m \cdot J_{eq1} + J_{Arm1} (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_1 / R_m \cdot J_{eq1}$$

where,

$$\ddot{\alpha}_1 = -K_{stiff} \cdot \alpha_1 \{ (1/J_{Arm1}) + (1/J_{eq1}) - (\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m) / R_m \cdot J_{eq1} + (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_2 / R_m \cdot J_{eq1} \} \quad (17)$$

For second link;

$$J_{eq2} \cdot \ddot{\theta}_2 - K_{stiff} \cdot \alpha_2 = [\eta_m \cdot \eta_g \cdot K_t \cdot K_g (V_m - K_g \cdot K_m \cdot \dot{\theta}_2)] / R_m$$

$$J_{eq1} \ddot{\theta}_2 = K_{stiff} \cdot \alpha_2 + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m] / R_m - [\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m \cdot \dot{\theta}_2] / R_m - B_{eq} \cdot \dot{\theta}_2$$

or,

$$\ddot{\theta}_2 = [K_{stiff} \cdot \alpha_2] / J_{eq1} + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m] / R_m \cdot J_{eq2} - [(\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_2] / R_m \cdot J_{eq2} \quad (18)$$

Put value of $\ddot{\theta}_2$ in equation (22);

$$J_{Arm2} \cdot \ddot{\theta}_2 + J_{Arm2} \cdot \ddot{\alpha}_2 + K_{stiff} \cdot \alpha_2 = 0$$

$$J_{Arm2} \ddot{\theta}_2 + J_{Arm2} \ddot{\alpha}_2 = -K_{stiff} \cdot \alpha_2$$

$$J_{Arm2} [K_{stiff} \cdot \alpha_2] / J_{eq2} + [\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m] / R_m \cdot J_{eq2} - (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_2 / R_m \cdot J_{eq2} + J_{Arm1} \ddot{\alpha}_2 = -K_{stiff} \cdot \alpha_2$$

or,

$$J_{Arm2} \cdot \ddot{\alpha}_2 = -K_{stiff} \cdot \alpha_2 [1 + (J_{Arm2} / J_{eq2})] - J_{Arm2} [(\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m) / R_m \cdot J_{eq2}] + J_{Arm2} (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_2 / R_m \cdot J_{eq2}$$

And therefore,

$$\ddot{\alpha}_2 = -K_{stiff} \cdot \alpha_2 \{ (1/J_{Arm2}) + (1/J_{eq2}) - (\eta_m \cdot \eta_g \cdot K_t \cdot K_g \cdot V_m) / R_m \cdot J_{eq2} + (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) \cdot \dot{\theta}_2 / R_m \cdot J_{eq2} \} \quad (19)$$

The mathematical model may be expressed in state space form. The state space model is expressed as matrix form, which is essential for the state space control method. For state space representation, the value of $\dot{\theta}_1, \dot{\theta}_2,$

$\ddot{\alpha}_1, \ddot{\alpha}_2$ is obtained from equation 16,17,18 and 19.

$$\dot{\theta}_1 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 + 0 \cdot \alpha_2 + 1 \cdot \dot{\theta}_1 + 0 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + 0 \cdot V_m \quad (20)$$

$$\dot{\theta}_2 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 + 0 \cdot \alpha_2 + 0 \cdot \dot{\theta}_1 + 1 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + 0 \cdot V_m \quad (21)$$

$$\dot{\alpha}_1 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 + 0 \cdot \alpha_2 + 0 \cdot \dot{\theta}_1 + 0 \cdot \dot{\theta}_2 + 1 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + 0 \cdot V_m \quad (22)$$

$$\dot{\alpha}_2 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 + 0 \cdot \alpha_2 + 0 \cdot \dot{\theta}_1 + 0 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 1 \cdot \dot{\alpha}_2 + 0 \cdot V_m \quad (23)$$

$$\ddot{\theta}_1 = 0 \cdot \theta_1 + 0 \cdot \theta_2 - k_{stiff} / J_{eq1} \cdot \alpha_1 + 0 \cdot \alpha_2 + A_1 \cdot \dot{\theta}_1 + 0 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + W \cdot V_m \quad (24)$$

$$\ddot{\theta}_2 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 - k_{stiff} / J_{eq1} \cdot \alpha_2 + 0 \cdot \dot{\theta}_1 + B_1 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + X \cdot V_m \quad (25)$$

$$\ddot{\alpha}_1 = 0 \cdot \theta_1 + 0 \cdot \theta_2 - k_{stiff} (1/J_{arm1} + 1/J_{eq1}) \cdot \alpha_1 + 0 \cdot \alpha_2 + C_1 \cdot \dot{\theta}_1 + 0 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + Y \cdot V_m \quad (26)$$

$$\ddot{\alpha}_2 = 0 \cdot \theta_1 + 0 \cdot \theta_2 + 0 \cdot \alpha_1 - k_{stiff} (1/J_{arm2} + 1/J_{eq2}) \alpha_2 + 0 \cdot \dot{\theta}_1 + D_1 \cdot \dot{\theta}_2 + 0 \cdot \dot{\alpha}_1 + 0 \cdot \dot{\alpha}_2 + Z \cdot V_m \quad (27)$$

Where, $A_1 = -(\eta_m \cdot \eta_g \cdot K_t \cdot K_m \cdot K_g^2 + B_{eq} \cdot R_m) / R_m \cdot J_{eq1}$

$B_1 = (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) / R_m \cdot J_{eq2}$

$C_1 = (\eta_m \cdot \eta_g \cdot K_t \cdot K_m \cdot K_g^2 + B_{eq} \cdot R_m) / R_m \cdot J_{eq1}$

$D_1 = (\eta_m \cdot \eta_g \cdot K_t \cdot K_g^2 \cdot K_m + B_{eq} \cdot R_m) / R_m \cdot J_{eq2}$

Matrix form can be obtained from equations 20,21,22,23,24,25,26 & 27.

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\alpha}_1 \\ \ddot{\alpha}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K_{stiff} \cdot \alpha_1}{J_{eq1}} & 0 & A_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{K_{stiff} \cdot \alpha_2}{J_{eq2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{stiff} [(1/J_{Arm1}) + (1/J_{eq1})] & 0 & 0 & C_1 & 0 & 0 \\ 0 & 0 & 0 & -K_{stiff} [(1/J_{Arm2}) + (1/J_{eq2})] & 0 & 0 & D_1 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \alpha_1 \\ \alpha_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq1}} \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq2}} \\ -\frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq1}} \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq2}} \end{bmatrix} V_m \quad (28)$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K_{stiff} \cdot \alpha_1}{J_{eq1}} & 0 & A_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{K_{stiff} \cdot \alpha_2}{J_{eq2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_{stiff} [(1/J_{Arm1}) + (1/J_{eq1})] & 0 & 0 & C_1 & 0 & 0 \\ 0 & 0 & 0 & -K_{stiff} [(1/J_{Arm2}) + (1/J_{eq2})] & 0 & 0 & D_1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq1}} \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq2}} \\ -\frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq1}} \\ \frac{\eta_m \cdot \eta_g \cdot K_t \cdot K_g}{R_m \cdot J_{eq2}} \end{bmatrix} V_m \quad (29)$$

$$C = [1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0] \quad D = [0]$$

Substituting the values of parameters of double link flexible manipulators in above equation 28 and 29, we obtain the following model:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 7.96 & 0 & 2.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.10 & 0 & -2.16 & 0 & 0 \\ 0 & 0 & -412.4 & 0 & 2.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -408.6 & 0 & 2.16 & 0 & 0 \end{bmatrix} \quad (30)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.29 \\ 1.39 \\ -1.29 \\ -1.39 \end{bmatrix}$$

$$C = [1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0] \quad D = 0$$

III. RESULTS AND DISCUSSION: Stability analysis and controller implementation of Flexible Link Manipulator

The Transfer function for double link flexible manipulator has been derived from its state space model i.e. using equation 38 and 39.

$$\theta(s) / V_m = G(s) = \frac{-45.25s^7 + 120.5s^6 + 6.984s^5 + 1131s^4 + 2472s^3 + 63.11s^2 - 60.7s + 28.1}{s^8 - .5475s^7 + 815.2s^6 - 418.7s^5 + 22.2s^4 - 86.6s^3 - 132.6s^2}$$

By using the root- Locus method, it is possible to determine the value of the loop gain K that will make the damping ratio of the dominant closed-loop poles as prescribed. If the location of an open-loop pole or zero is a system variable, then the root-locus method suggests the way to choose the location of an open- loop pole or zero. The root locus plot of above transfer function is shown in figure 2.

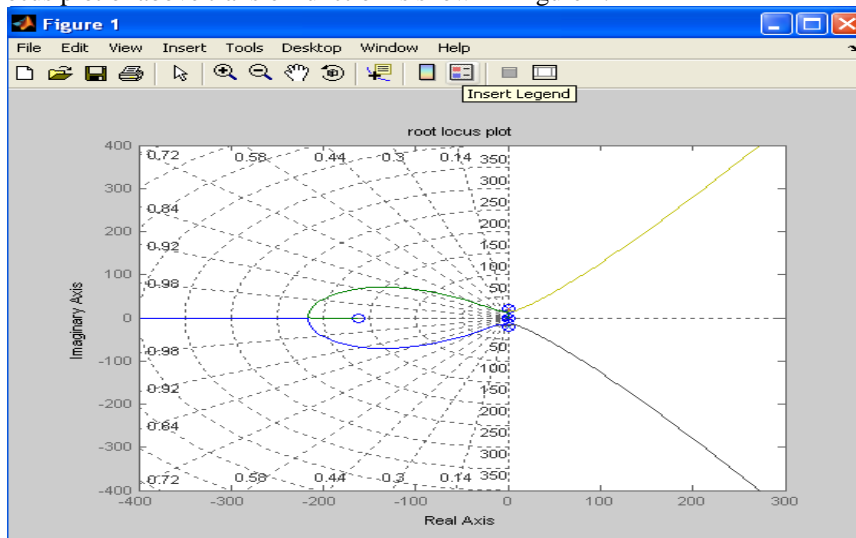


Figure 2: Root Locus Plot for double link Flexible manipulator

It is clear from figure 2 that the system is unstable. The Nyquist and bodeplot of above transfer function has been illustrated in figure 3 and figure 4 respectively. As it is clear from above figure 3 and figure 4 the gain margin and phase margin are negative. Hence the system is highly unstable.

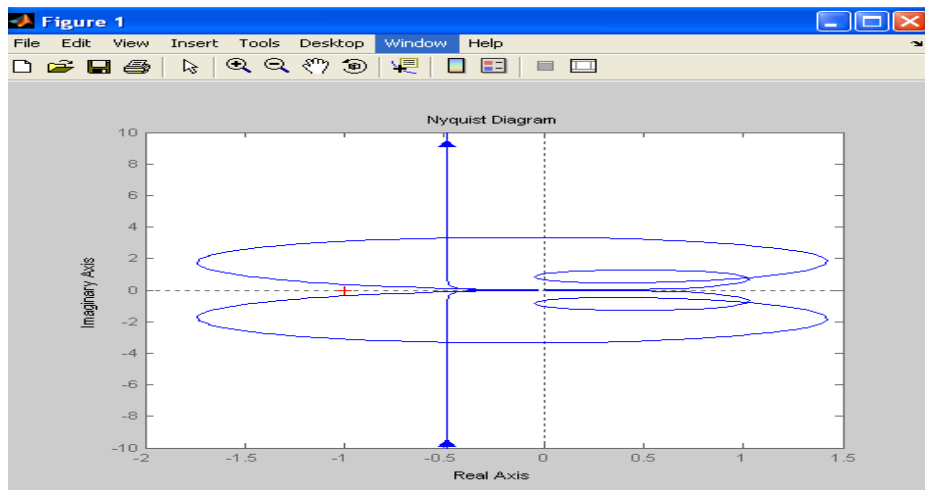


Figure 3: Nyquist plot for double link Flexible manipulator

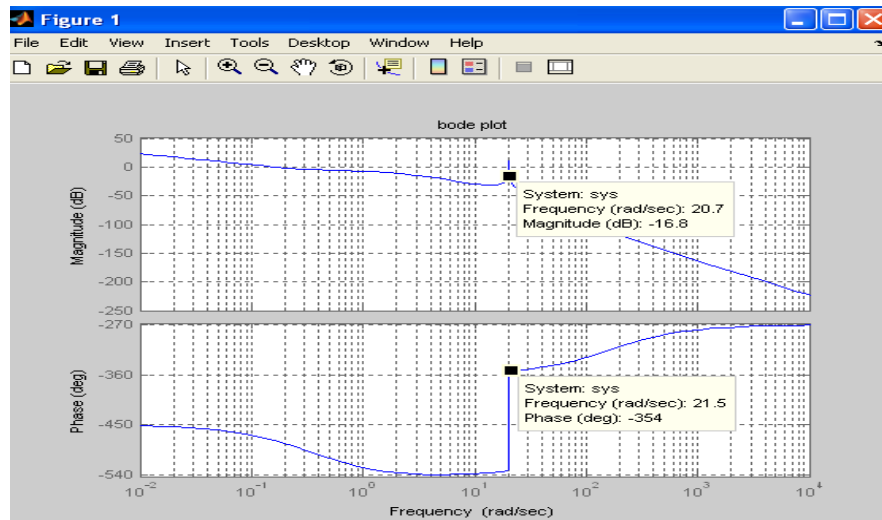


Figure 4 : Bode plot for double link Flexible system

Controller Implementation of Flexible Link Manipulator: A controller compares the actual value of the plant output with the reference input, determines the deviation [16] and produces a control signal that will reduce the deviation to zero or to a small value. The equation of a PID controller is given by-

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$$

$$U(s) = [K_p + K_i \frac{1}{s} + K_d s]E(s)$$

$$\frac{U(s)}{E(s)} = \frac{K_d s^2 + K_p s + K_i}{s}$$

(31)

Tuning of a PID involves the adjustment of K_p , K_i , and K_d to achieve some user defined "optimal" character of system response.

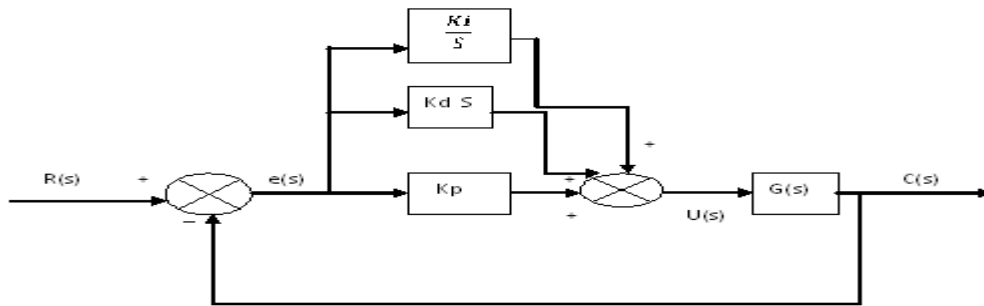


Figure 5: PID Controller of Flexible link system

Where,

K_p = Proportional Gain ; K_d = Derivative Gain; K_i = Integral Gain; $R(s)$ = reference signal
 $G(s)$ = Transfer Function of flexible link manipulator; $e(s)$ = error signal; $u(s)$ = plant input
 $y(s)$ = output from the plant

Therefore, the open-loop transfer function of the diagram can be found as shown by the equation:

$$Y(s)/U(s) = G(s) * [K_p + K_d s + (K_i/s)] \tag{32}$$

Where $G(s)$ is the plant transfer function.

The open-loop transfer function in equation (32) can be implemented into Matlab by using the m-file code. The transfer function from the Laplace transforms or from the state space representation can be used. This transfer function assumes that both derivative and integral control will be needed along with proportional control. This open-loop transfer function can be modeled in Matlab. The function polyadd is not originally in the Matlab toolbox. It has to be copied to a new m-file to use it. This transfer function is assumed that both derivative and integral control will be needed along with proportional control. The actual control of this system could be stated by subjecting step input and its response which has been shown in figure 6.



Figure 6: Closed loop PID Response of double link flexible manipulator

State Feedback Controller

The pole placement approach specifies all closed-loop poles in comparison to conventional design approach which specifies only dominant closed loop poles [8]. In pole-placement approach the desired closed-loop poles can be arbitrarily placed, provided the plant is completely state controllable. In designing a system using the pole placement approach several different sets of desired closed-loop poles need to be considered, the response characteristics compared, and the best one chosen.

Consider a control system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{33}$$

Where, x = state vector (n -vector); y = output signal (scalar)

u = control signal (scalar); $A = n \times n$ constant matrix

$B = n \times 1$ constant matrix; $C = 1 \times n$ constant matrix

D = constant (scalar)

Let choose the control signal to be.

$$u = -Kx \tag{34}$$

This means that the control signal u is determined by an instantaneous state. Such a scheme is called state feedback. The $1 \times n$ matrix K is called the state feedback gain matrix. A block diagram for this system is shown in figure 7.

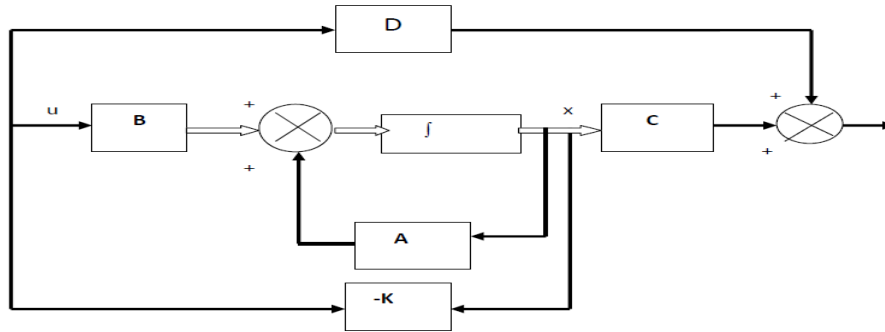


Figure 7: Design of control system in state space

Substituting Equation (34) into equation (33) gives

$$\dot{x}(t) = (A-BK) x(t)$$

The solutions of this equation is given by

$$x(t) = e^{(A-BK)t} x(0) \tag{35}$$

Where $x(0)$ is the initial state caused by external disturbances. The stability and transient response characteristics are determined by the Eigen values of matrix $A-BK$. If matrix K is chosen properly, the matrix $A-BK$ can be made an asymptotically stable matrix, and for $x(0) \neq 0$, it is possible to make $x(t)$ approach 0 as t approaches infinity. The eigen-values of matrix $A-BK$ are called the regular poles. If these regulator poles are placed in the left-half s plane, then $x(t)$ approaches 0 as t approaches infinity. The problem of placing the regular poles (closed-Loop poles) at the desired location is called a pole- place-placement problem.

Pole-placement problems can be solved easily with MATLAB. MATLAB has two commands- **acker** and **place** – for the computation of feedback gain matrix K . The command **acker** is based on Ackerman’s formula. This command applies to single-input system only. [17]. The most commonly used commands are

$$K = \text{acker}(A,B,J)$$

or,

$$K = \text{place}(A,B,J)$$

where A and B are system matrices and J is a row vector containing the desired closed-loop poles. These commands returns the gain vector K .

CASE 1

Choose the desired closed -loop poles at

$$-2+j*4, -2-j*4, -3+2*j, -3-2*j, -6, -2, -10, -4$$

A unit step response of flexible double link system using feedback gain matrix (K_o) from pole-placement method is shown in figure 8.

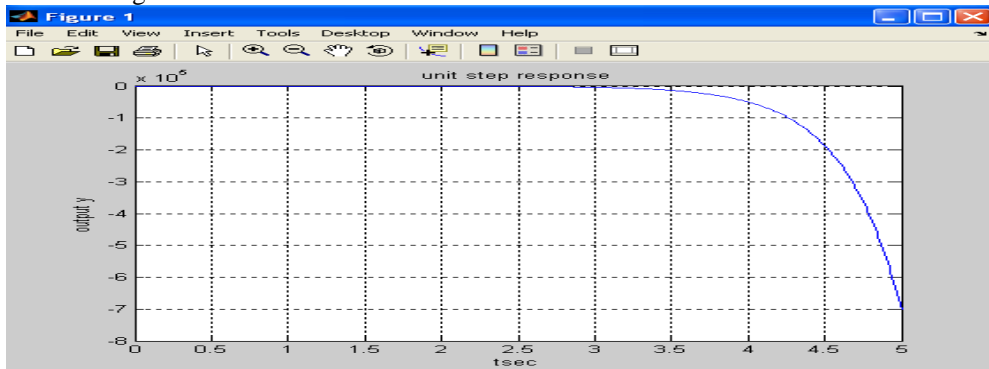


Figure 8: Unit step response of double link of flexible manipulator using Ackerman method for case1

CASE 2

Choose the desired closed -loop poles at

$$-3+2j, -3-2j, -4+2j, -4-2j, -3, -2, -1, -4$$

A unit step response of flexible double link system using feedback gain matrix (K_o) from pole-placement method is shown in figure 9.

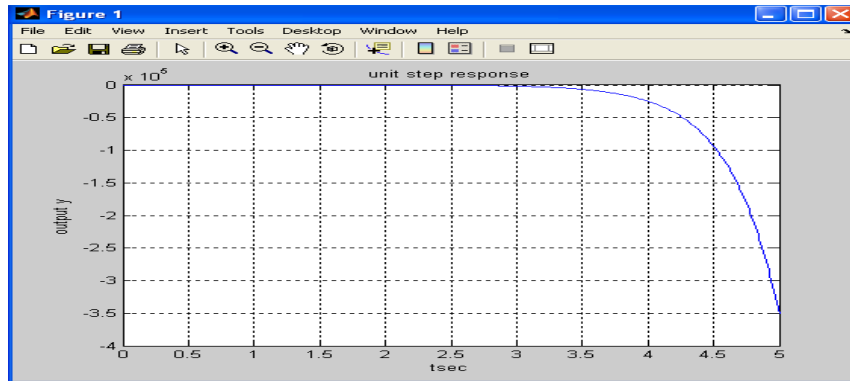


Figure 9: Unit step response of double link of flexible manipulator using Ackerman method for case2

Linear Quadratic Regulator

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called the LQ problem. One of the main results in the theory is that the solution is provided by the **linear-quadratic regulator (LQR)**. The "cost" (function) is often defined as a sum of the deviations of key measurements from their desired values. In effect this algorithm therefore finds those controller settings that minimize the undesired deviations, like deviations from desired altitude or process temperature [17]. Often the magnitude of the control action itself is included in this sum as to keep the energy expended by the control action itself limited. The LQR algorithm is, at its core, just an automated way of finding an appropriate state feedback controller. Consider the optimal regulator given by system equation:

$$\dot{x} = Ax + Bu \tag{36}$$

That determines the matrix K of optimal control vector

$$u(t) = -Kx(t) \tag{37}$$

So as to minimize the performance index

$$J = \int_0^{\infty} (x^* Qx + u^* Ru) dt \tag{38}$$

Where, Q and R is a positive definite or real symmetric matrix. The block diagram showing the optimal configuration is shown in figure 10.

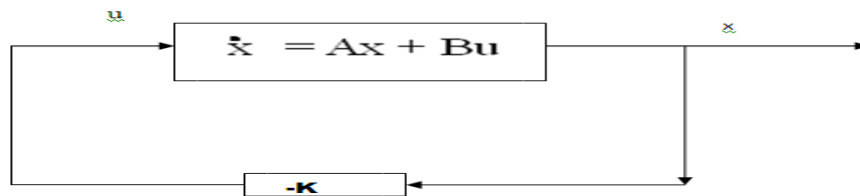


Figure 13 Quadratic Optimal Regulator System

Substituting equation (37) into (36), we obtain

$$\dot{x} = Ax - BKx = (A-BK)x$$

Substituting equation (37) into (38), we obtain

$$J = \int_0^{\infty} (x^* Qx + x^* K^* R K x) dt$$

Let us set

$$x^*(Q + K^* R K)x = -d/dt (x^* P x)$$

Where, P is positive definite Hermitian-Matrix.

$$x^*(Q + K^* R K)x = -x^*[(A-BK)^* P + P (A-BK)]x$$

Comparing both sides of this equation and notice that this equation must hold true for any x, we require that $(A-BK)^* P + P(A-BK) = -(Q + K^* R K)$ (39)

It can be proved that if A-BK is a stable matrix, there exist a [8] positive definite matrix that satisfies equation (39).

The performance index J can be evaluated as

$$J = \int_0^{\infty} (x^*Qx + x^*K^*RKx)dt = x^*(\infty)Px(\infty) + x^*(0)Px(0) \quad (40)$$

Since all Eigen values of A-BK are assumed to have negative real parts, we have $x(\infty) \rightarrow 0$.

Therefore, we obtain $J = x^*(0)Px(0)$

Thus, the performance index J can be obtained in terms of initial condition $x(0)$ and P.

To obtain the solution of quadratic optimal control problem, we proceed as follows

$R = T^*T$ Where, T is a non-singular matrix. Then equation (39) can be written as $(A^* - K^*B^*)P + P(A - BK) + Q + K^*T^*TK = 0$.

The minimization of J w.r.t. K requires the minimization of

$$x^*[TK - (T^*)^{-1}B^*P]^*[TK - (T^*)^{-1}B^*P]x \text{ w.r.t. } K$$

The minimum occurs when it is zero or when

$$TK = (T^*)^{-1}B^*P$$

$$\text{Hence, } K = T^{-1}(T^*)^{-1}B^*P = R^{-1}B^*P \quad (41)$$

Equation (41) gives the optimal matrix K. Thus, the optimal control law to the quadratic optimal control problem, when the performance index is given by equation (38) is linear and is given by

$$u(t) = -Kx(t) = R^{-1}B^*Px(t). \text{ The matrix P in equation (41) must satisfy equation (39) or the following}$$

reduced equation:

$$A^*P + PA - PB R^{-1}B^*P + Q = 0 \quad (42)$$

Equation (42) is called the reduced – matrix Riccati equation. The design steps may be stated as follows:

- i. Solve equation (42), the reduced – matrix Riccati equation, for matrix P.
- ii. Substitute this matrix P into equation (41). The resulting matrix K is the optimal matrix.

The Unit step response of double link flexible manipulator using LQR method is illustrated in figure14.

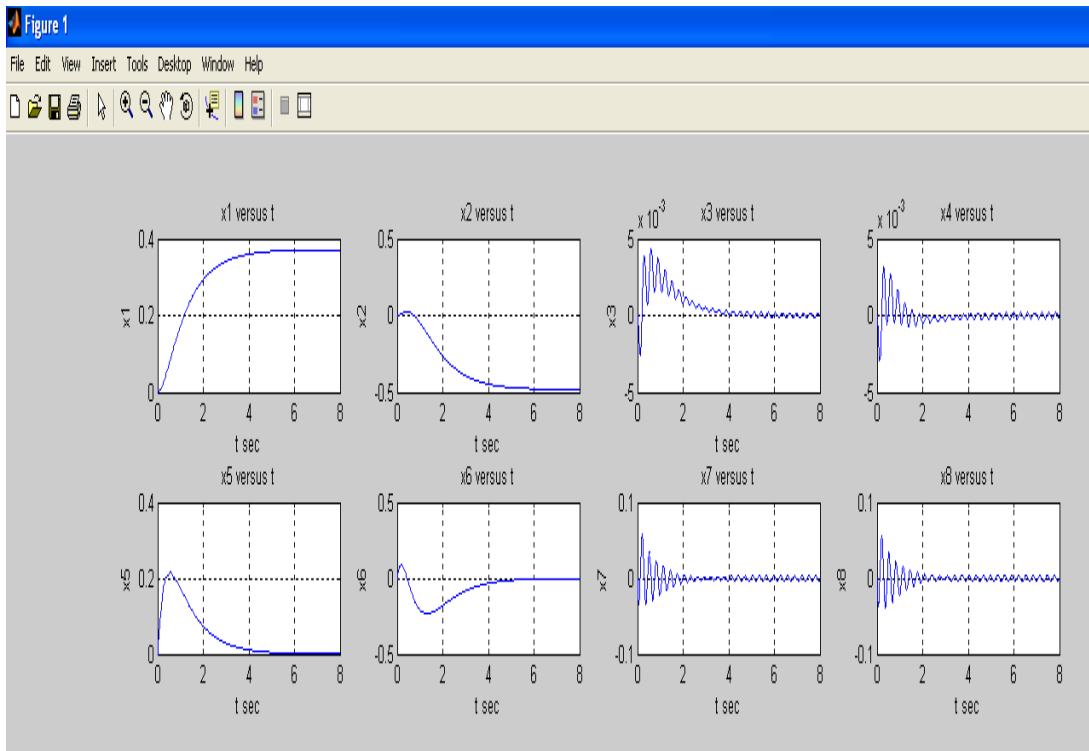


Figure 14 : Unit step response of double link flexible manipulator using LQR method

Where, x_1 corresponds to θ_1 x_2 corresponds to θ_2 x_3 corresponds to α_1
 x_4 corresponds to α_2 x_5 corresponds to $\dot{\theta}_1$ x_6 corresponds to $\dot{\theta}_2$
 x_7 corresponds to $\dot{\alpha}_1$ x_8 corresponds to $\dot{\alpha}_2$

IV. CONCLUSIONS

The various aspects on mathematical modelling, stability and control strategies of flexible double link robotic manipulator have been investigated in this paper. A mathematical model of flexible double link manipulator has been developed using lagrangian method. Mathematical model of flexible double link robotic manipulator have been characterized using classical and modern control theories and its time domain and frequency domain analysis has been carried out. Our study shows that the mathematical model of flexible manipulator is highly unstable system. Different control strategies such as PID, LQR and State feedback controller have been implemented for controlling the tip position of flexible double link manipulator. State feedback controller uses pole placement approach, while the linear quadratic regulator is obtained by resolving the Riccati equation.

It was concluded from our results of implementation of various controllers for flexible double link manipulator problem that the state feedback and linear quadratic regulator (LQR) controllers providing the better performance than conventional PID controllers. The performance of state feedback controller is depending upon pole positions and hence it becomes hit and trial procedure for controlling the flexible double link manipulator. The best control strategy for controlling the tip position of flexible double link manipulators is obtained by implementation of LQR controller. Finally it can be conclude from our study that LQR control method is the best method among PID and State feedback controller to control the flexible double link manipulator.

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